

Coverage and lacunarity of astronomical datasets



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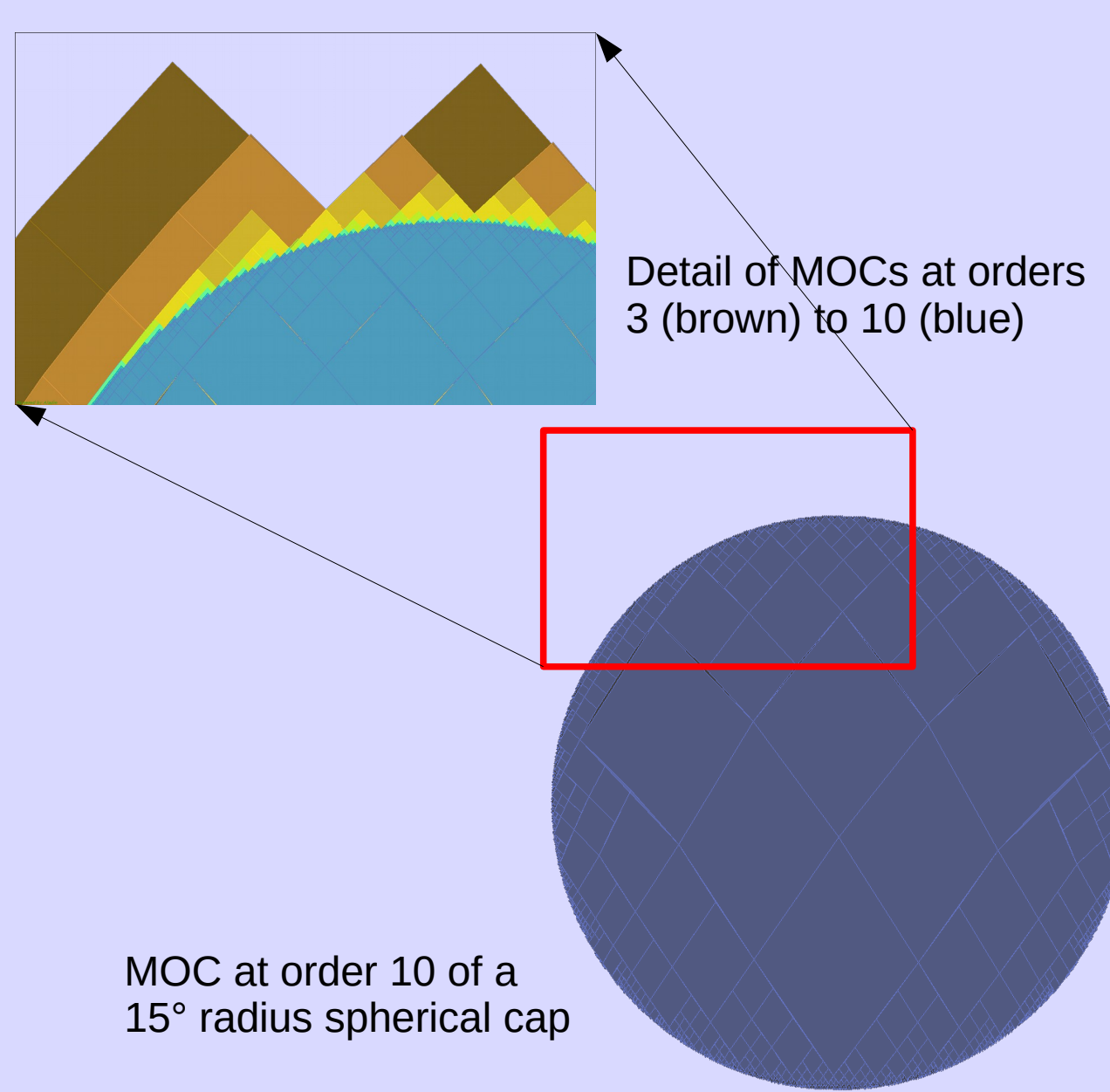
Abstract

We present the use of MOCs (Multi-Order Coverage maps) associated to HiPS datasets (Hierarchical Progressive Surveys) to assess the sky coverage of various datasets (image surveys and source catalogues), but also their lacunarity (presence of holes).

The goal is to :

- Estimate at best the sky coverage of a dataset
- Find a metric to quantify the scattering of data on the sky (distinguish contiguous datasets from patches of data scattered over large areas, or containing many holes)
- Apply this to source catalogues, without oversampling

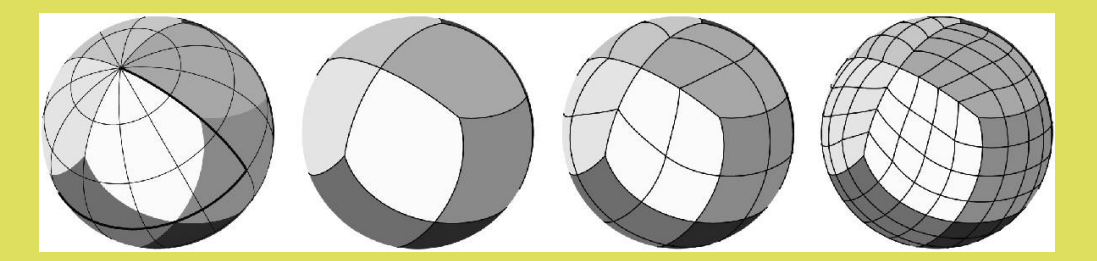
MOCs of mock data



- MOCs can be used to estimate the sky coverage, by adding the areas of all HEALPix pixels.
- The total area is overestimating the true area, because pixels spread on the perimeter.
- The highest the order of the MOC, the better the precision.
- The area of a HEALPix pixel at order k is $\Omega_{\text{pix}} = (\pi/3) 4^{-k}$, and the estimated area from a MOC at order k is related to the true area by a relation of the form $S_k = S_{\text{true}} (1 + L \cdot 2^{-k})$ where L is a parameter related to the perimeter of the MOC.
- More complex MOCs will have larger values of L .
- We define $\mathcal{L} = \log(L)$ as the **lacunarity** of the dataset : small values will indicate simple and contiguous data distribution, while large values will characterize scattered or irregular data.

The notion of lacunarity was introduced by Mandelbrot to describe the texture of fractals, and was extended (see for example Dong (2000)) to describe the distribution of gap sizes in geometric objects : homogeneous objects have low lacunarity, and heterogeneous objects have high lacunarity.

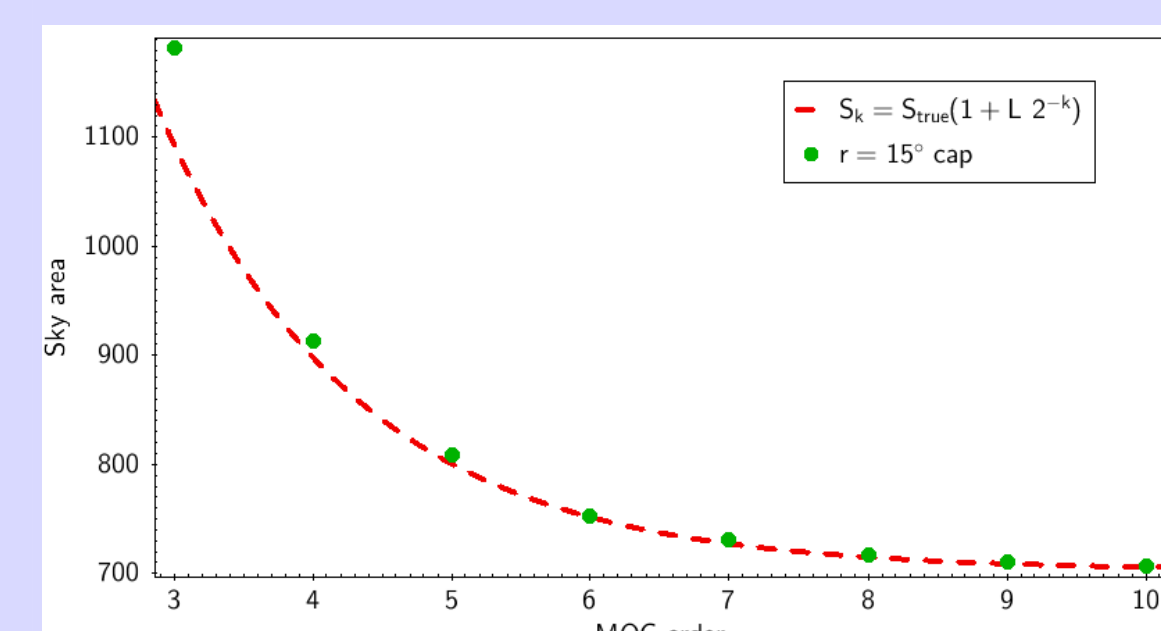
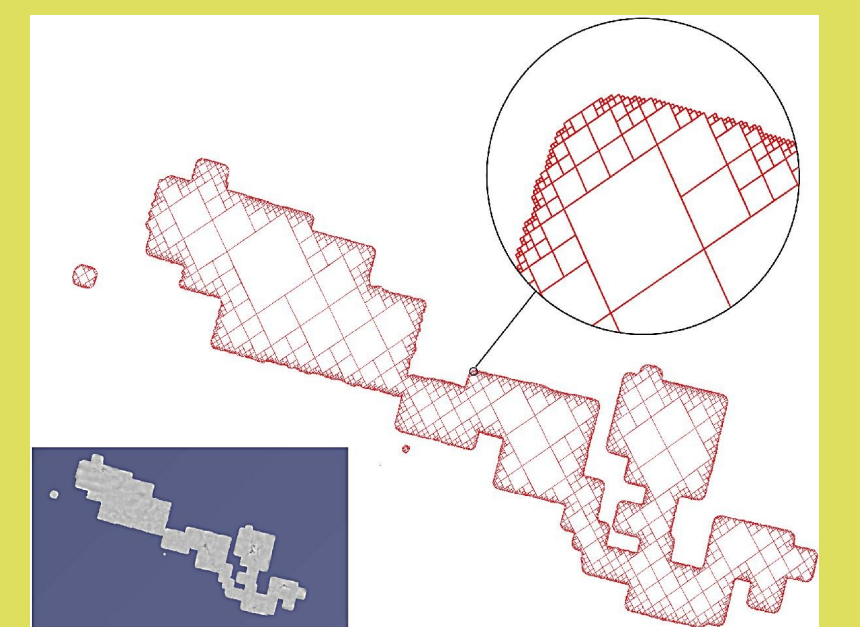
HiPS and MOCs



See Fernique et al. (2015)

- Based on the HEALPix tessellation of the sphere (equal area)
- Progressive surveys (HiPS) and coverage maps (MOCs) allow to describe complex datasets
- Precision increases with order k

k	$N_{\text{pix}} = 2^{2k}$	N_{pix}	θ_{pix}	R_{HealPix}	$N_{\text{MOC},k}$	$\theta_{\text{MOC},k}$
0	1	12	58.6			
1	2	48	29.3			
2	4	192	14.7			
3	8	768	7.33			
4	16	3072	3.66			
5	32	12288	1.83			
6	64	49152	0.91			
7	128	196608	0.45			
8	256	786432	0.23			
9	512	3145728	0.11			
10	1024	12582912	0.05			
11	2048	50331648	0.02			
12	4096	201326592	0.01			
13	8192	805306368	0.005			
14	16384	3221226432	0.002			
15	32768	12884905728	0.001			
16	65536	51539622912	0.0005			
17	131072	206158491648	0.0002			
18	262144	824633966592	0.0001			
19	524288	3300535466368	5e-05			
20	1048576	13202141865472	2.5e-05			
21	2097152	52808567461888	1.25e-05			
22	4194304	211234269847552	6.25e-06			
23	8388608	844937079390144	3.125e-06			
24	16777216	3383748317560576	1.5625e-06			
25	33554432	13535033270242304	7.8125e-07			
26	67108864	54032133088969216	3.90625e-07			



Computing the MOCs at different orders k for a $r=15^\circ$ spherical cap, and fitting the formula over orders $k=7$ to 10 gives an estimated $S_{\text{true}} = 704.5 \text{ deg}^2$, very close to the exact area $S=702.8 \text{ deg}^2$, with $\mathcal{L} = 0.648$.

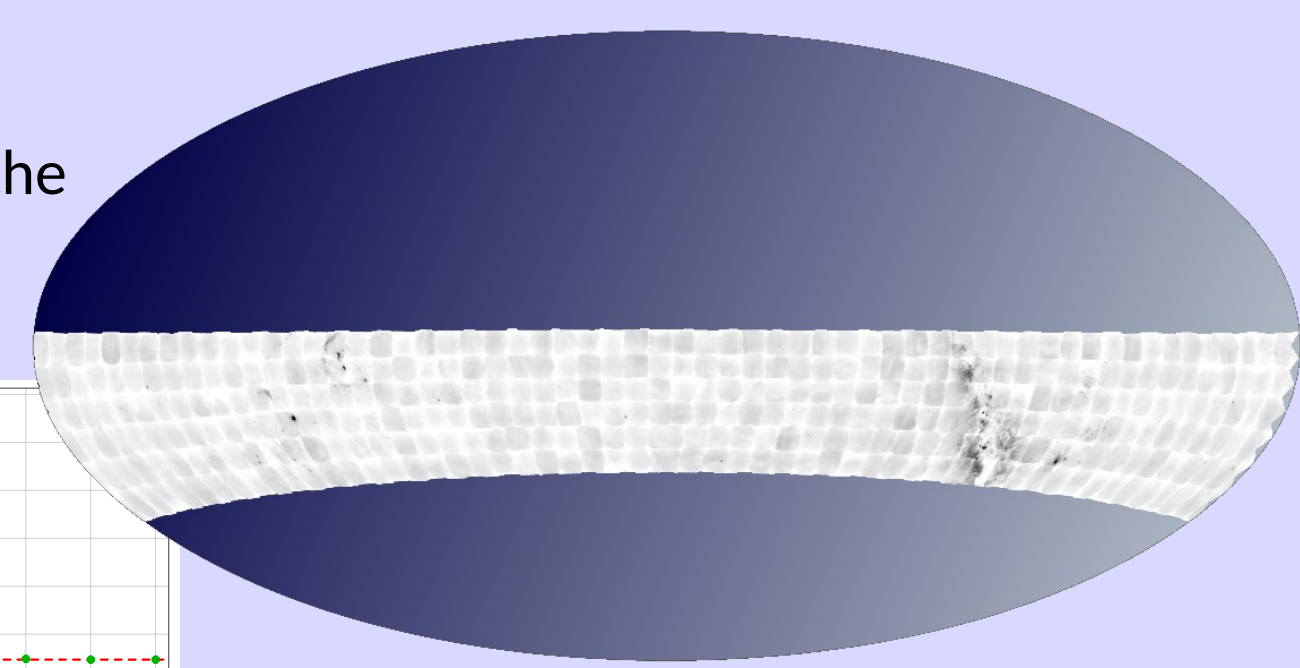
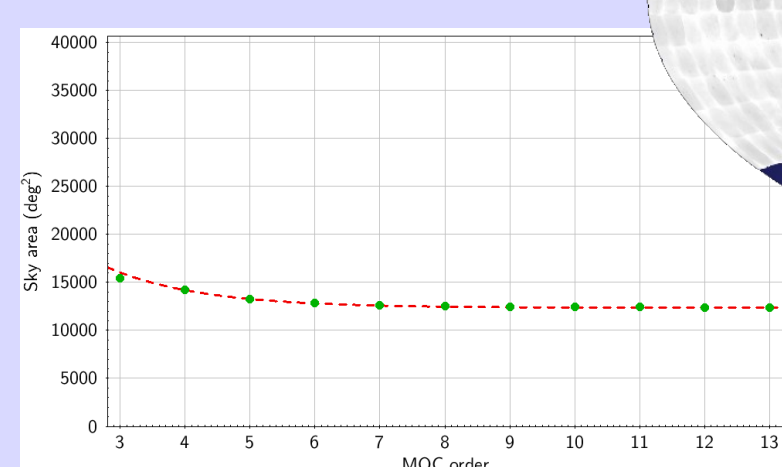
For a MOC corresponding to 3 disjoint caps, the fit yields $S_{\text{true}} = 2109.4 \text{ deg}^2$, very close to the exact area $S=2108.4 \text{ deg}^2$, with $\mathcal{L} = 0.715$.

Application to image HiPS

For 3 example image HiPS, we compute MOCs from order $k=3$ down to a maximum order, and study the evolution of the MOC area with k .

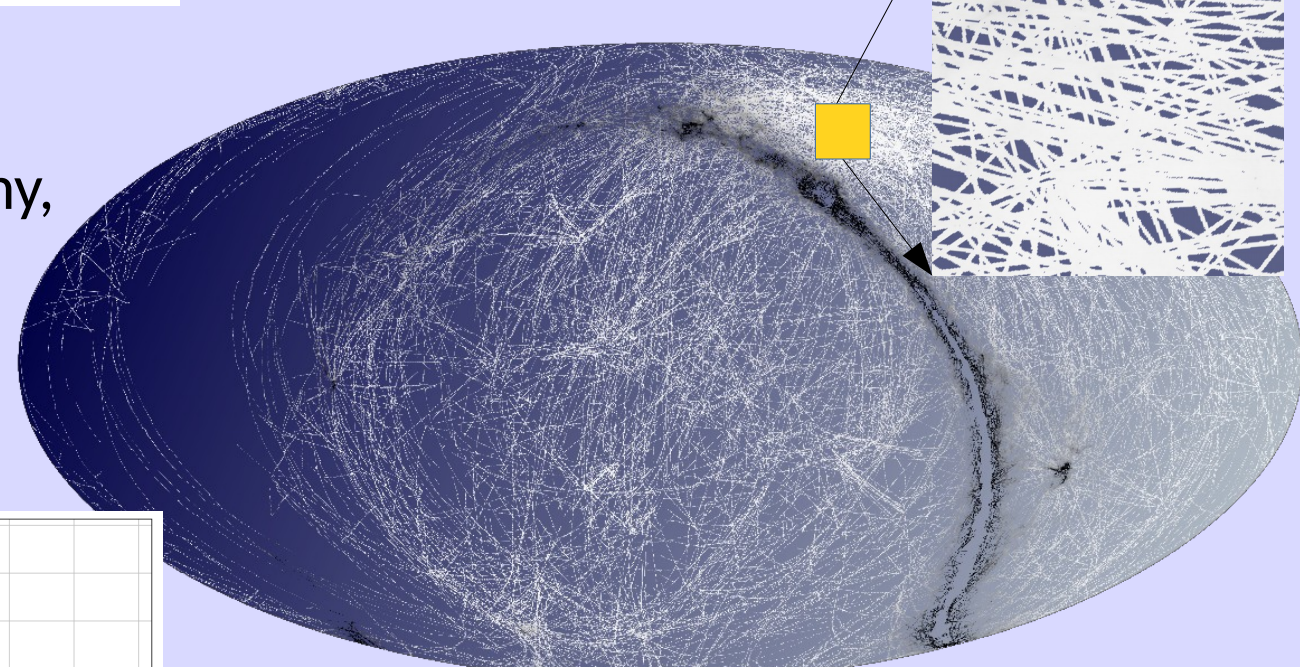
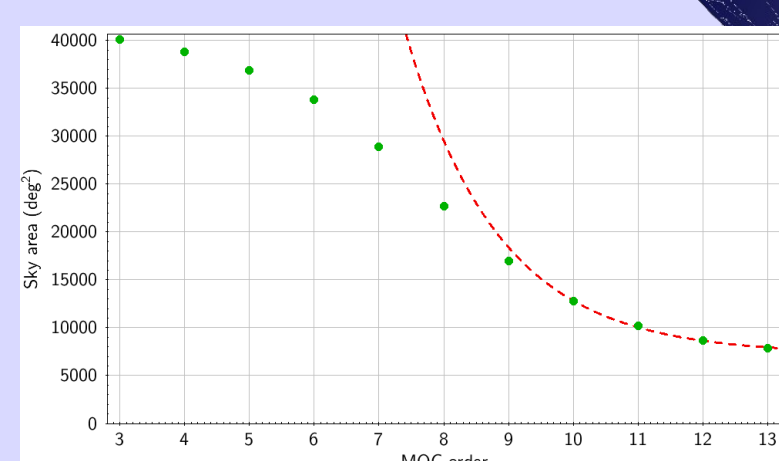
CDS/P/MAMA/posse

This survey corresponds to a contiguous fraction of the sky, with a simple boundary. Areas derived from MOCs quickly converge to the corresponding sky coverage.



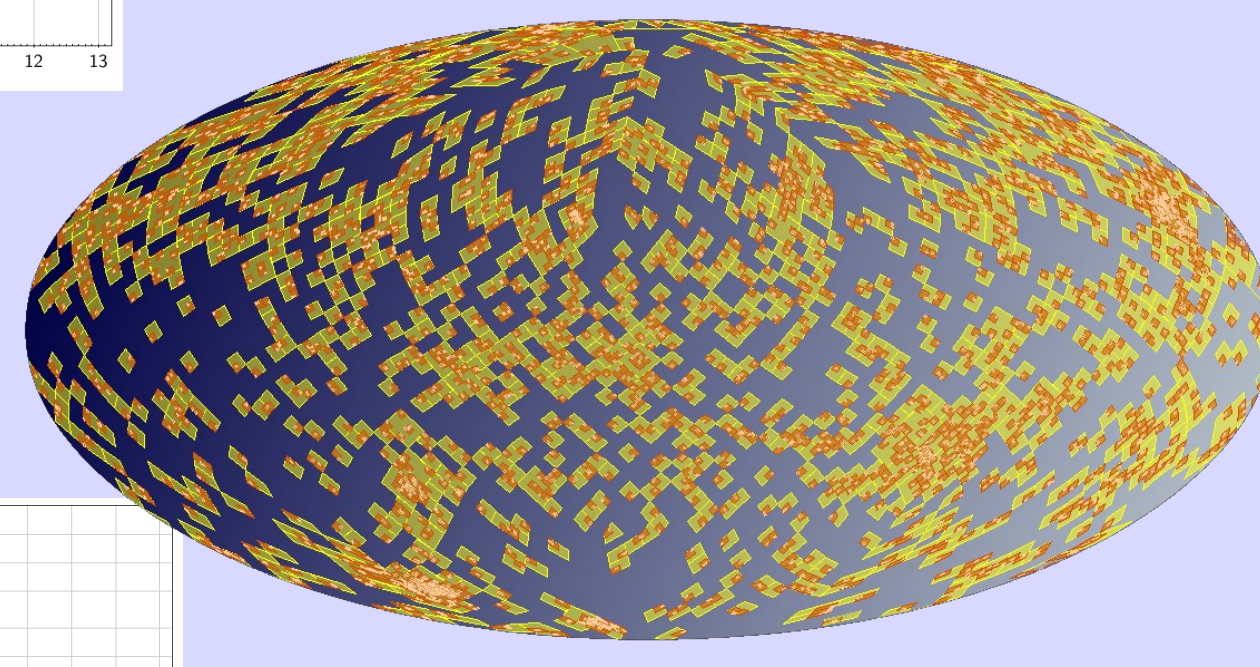
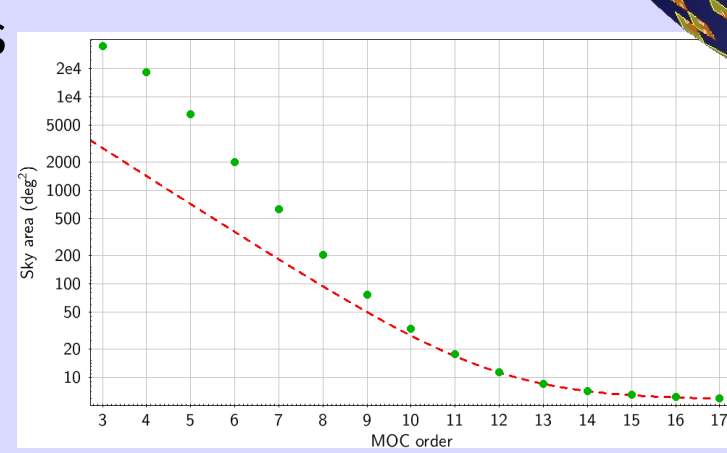
CDS/P/ISOPHOT/170

The ISOPHOT 170um serendipity survey is very patchy, with many unobserved holes in the slew survey. At low orders, the MOC pixels merge and the total MOC area becomes nearly all-sky. It is possible to fit the above law for $k=10$ to 13.



CDS/P/HST/V

This compilation of HST observations around the V band contains many small-area pointed observations scattered over the sky, as can be seen from the MOCs at order 5, 7 and 9 represented here. The MOC area quickly decreases with k , and we need to reach order $k=17$ to start converging towards the true coverage.



Lacunarity increases for heterogeneous datasets.

Results :

Comparing the coverages obtained here to the ones quoted in the CDS MOC server, one sees that the `moc_sky_fraction` for the ISOPHOT survey, computed at $k=10$, is overestimated.

Survey	S_{true} (deg ²)	\mathcal{L}	MOC Server (deg ²)
MAMA/posse	12358	0.377	12425
ISOPHOT	7252	2.896	12771
HST/V	5.745	3.595	5.978

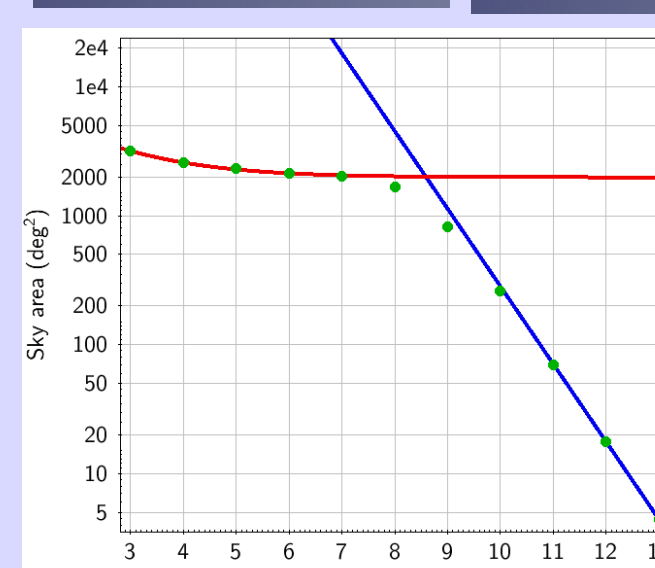
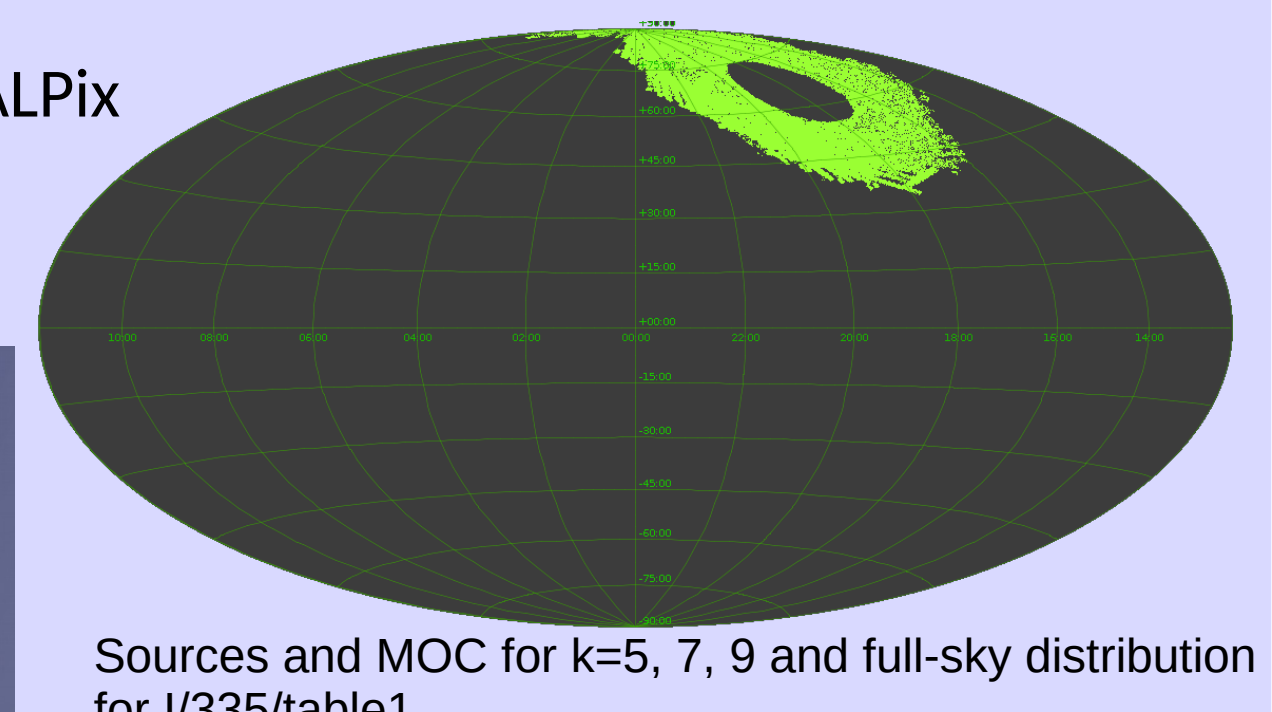
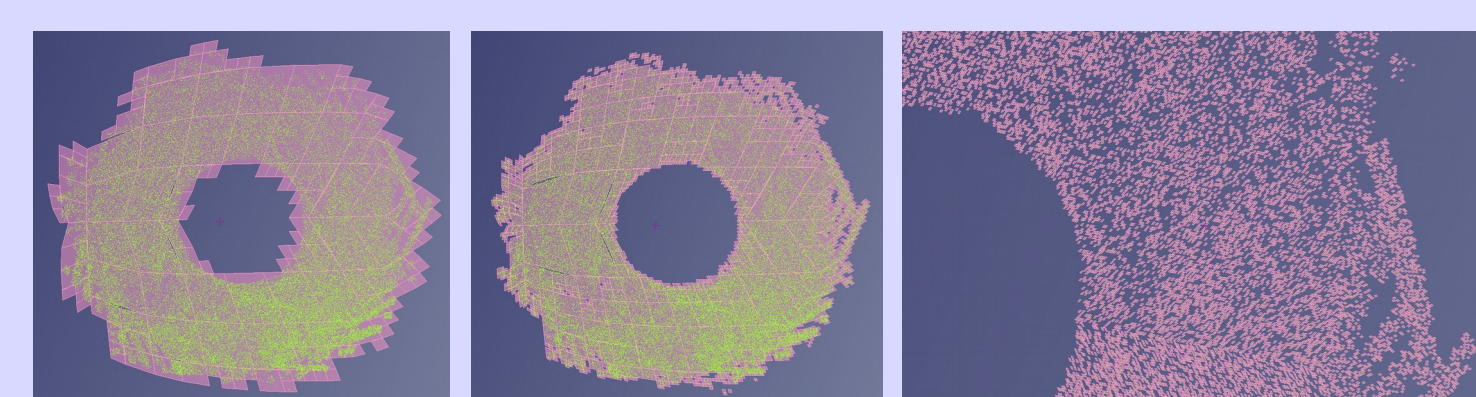
Application to catalogues

Estimating the sky coverage of a catalogue from the positions of its individual sources is more difficult than for an image survey : if the density of sources per HEALPix pixel becomes too low at high orders k , empty pixels cause an underestimation of the area.

To illustrate this, we study here two VizieR catalogues with a similar number of sources, but very different distributions on the sky.

CDS/I/335/table1 : LUT Survey Catalogue Data Release 1 (Men et al. 2016)

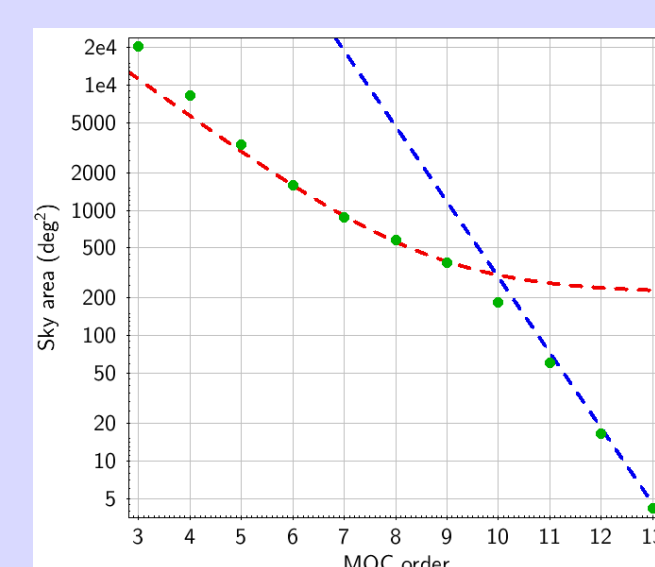
$N=86\ 467$ sources. When k is too large, there is a single source per HEALPix pixel, and the MOC area becomes $S_k = N(\pi/3) 4^{-k}$ (blue dashed line in the graphs).



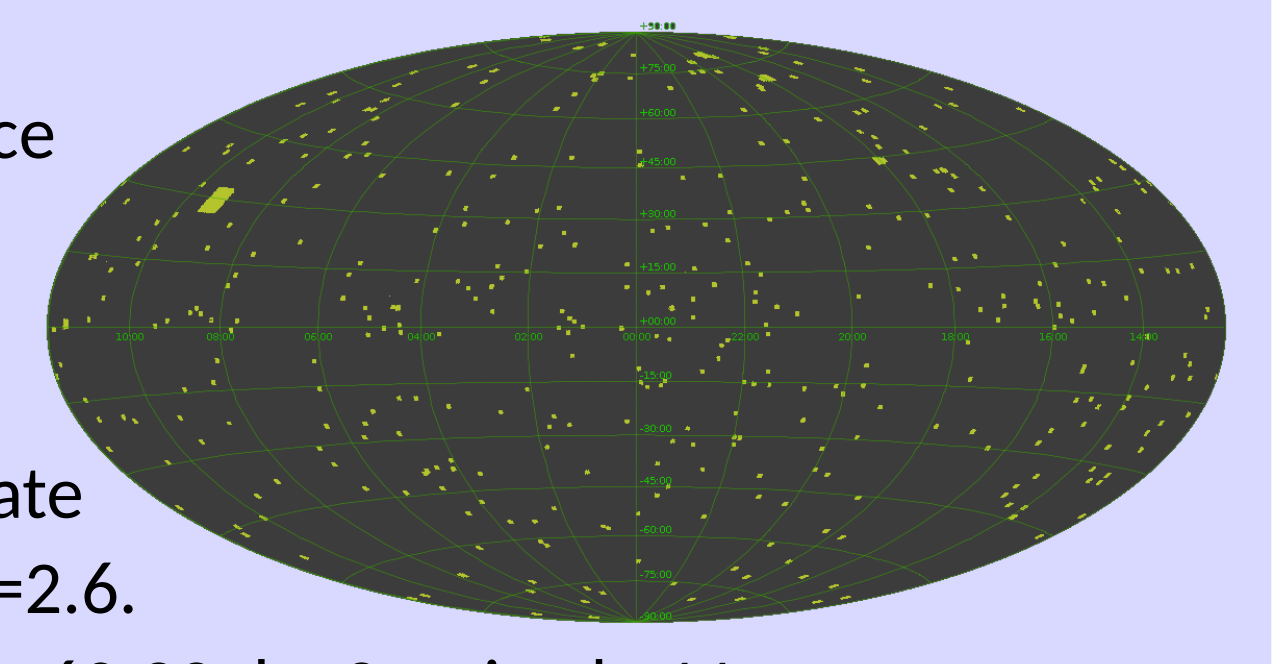
$k=3$ to 6 allow to estimate $S_{\text{true}} = 1984 \text{ deg}^2$ and $\mathcal{L}=0.683$. For $k>7$, empty pixels in the MOC give a wrong coverage. The MOC server gives $S=69.39 \text{ deg}^2$, using $k=11$.

CDS/I/273A/erlcat : Extragalactic Reference Link Catalog (de Vegt et al., 2001)

$N=89\ 422$ sources.



$k=6$ to 9 allow to estimate $S_{\text{true}} = 219.5 \text{ deg}^2$ and $\mathcal{L}=2.6$. The MOC server gives $S=60.39 \text{ deg}^2$, using $k=11$.



Conclusions: MOCs can provide a very convenient way to estimate the sky coverage of image surveys or catalogues. The analysis of the variations of the coverage with the MOC order also provide information on the scattering within the dataset, for example using the lacunarity introduced in this study.